TENTAMEN

Fontys
Hogescholen

Vak: MAT3 test exam

Naam student en studentnummer: Dos

Studierichting en klas: SE

Cijfer:

Datum: 15-10-2009

Aantal ingeleverde bladen:

Bladnr.:

Duidelijk invullen

1. $U = \{1, \cdots, 9\}$, $A = \{1,3,7,8,9\}$, $B = \{1,2,3,4\}$, $C = \{2,3,5,6,7\}$

a) $(A \cap B)^c$?

$A \cap B = \{1,3\}$, $(A \cap B)^c = \{2, 4, \cdots, 9\}$

b) $(A \cup B) \setminus C$?

$A \cup B = \{1,2,3,4,7,8,9\}$

$(A \cup B) \setminus C = \{1,4,8,9\}$

c) $B \oplus C = \{1,4,5,6,7\}$

2. X and Y are given twosets

a) Proof that $(X \setminus Y) \cap (X \cap Y) = \emptyset$, the proof consists of 2 parts

1. suppose $x \in X \setminus Y$, than $x \in X$ but $x \notin Y$, thus $x \notin X \cap Y$

2. suppose $x \in X \cap Y$, than $x \in X$ and $x \in Y$, thus $x \notin X \setminus Y$

b) Proof that $(X \setminus Y) \cup (X \cap Y) \subset X$,

suppose $x \in (X \setminus Y) \cup (X \cap Y)$, than $x \in X \setminus Y$ or $x \in X \cap Y$.

If $x \in X \setminus Y$ than $x \in X \wedge x \notin Y$, if $x \in X \cap Y$ than $x \in X \wedge x \in Y$. So in either case $x \in X$

3. $A = \{a,b,c\}$, $B = \{1,2,3\}$, $C = \{2,3,4\}$

Determine $(A \times B) \cap (A \times C)$

$A \times B = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3),(c,1),(c,2),(c,3)\}$

$A \times C = \{(a,2),(a,3),(a,4),(b,2),(b,3),(b,4),(c,2),(c,3),(c,4)\}$

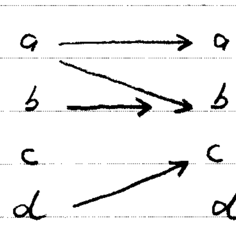
$(A \times B) \cap (A \times C) = \{(a,2),(a,3),(b,2),(b,3),(c,2),(c,3)\}$

4. $A = \{a,b,c,d\}$ $R = \{(a,a),(a,b),(b,b),(d,c)\}$

a)

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

b)



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Duidelijk invullen

c) $M_R \cdot M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) $R^{-1} = \{(a,a), (b,a), (b,b), (c,d)\}$

e) domain $(R) = \{a, b, c\}$
range $(R) = \{a, b, d\}$

f) $R \circ R = \{(a,a), (a,b), (b,b)\}$

5. $A = \{a, b, c\}$. $R = \{(a,a), (b,b), (c,c)\}$ and
$S' = \{(a,b), (b,a), (c,c)\}$

	R	S
reflexive	yes	no
symmetric	yes	yes
transitive	yes	no
anti-sym.	yes	no

6 $|A| = 3$, R and S' relations $A \to A$
Suppose $A = \{1,2,3\}$,
a) and R and S symetric than $R \cap S$ sym.
if $(a,b) \in R \cap S$, $(a,b) \in S$ and $(a,b) \in R$,
because R and symmetric $(b,a) \in S'$ and
$(b,a) \in R$, thus $(b,a) \in R \cap S'$, and thus
$R \cap S$ symmetric
b) $R = \{(1,2)\}$ $S' = \{(2,3)\}$ both
transitiv, but $R \cup S = \{(1,2), (2,3)\}$ is
not transitiv

7 R x fellow countryman of y
a) R is a equivalence relation because
R is reflexive (is given, xRx is possible)
R is sym. : xRy then yRx
R is transitive : xRy, yRz then xRz
b) Länder

8. f, g, h relations on $X = \{1,2,3,4\}$

$f = \{(1,2), (1,3), (2,2), (3,4), (4,3)\}$

$g = \{(1,4), (3,1), (2,3), (4,2)\}$

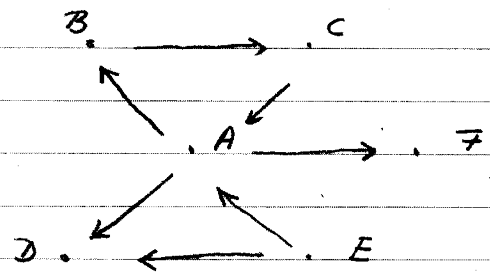
$h = \{(1,2), (2,3), (4,1), (3,1)\}$

a) g, h

b) g

c) g

9.



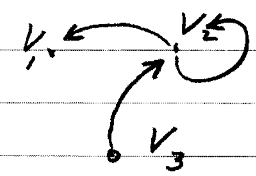
a) ABCABCA

b) EDABCAF is a spanning semipath, because only DA is no valid direction while AD valid is

10. $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad V = \{v_1, v_2, v_3\}$ $\quad G(V,E)$

G is directed

a)



b) $A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

12. $A = \{a, b\}$, $L = \{ b^k a b^l a b^m \mid k \geq 0, l \geq 0, m \geq 0 \}$

13. $A = \{a, b\}$, $L = \{ a^m b^n \mid m > 0, n > 0 \}$

$r = aa^*bb^*$

14. $L(G) = \{ a^n c b^n \mid n > 0 \}$ find context free G

$S \to aAb$

$A \to aAb$, $A \to c$

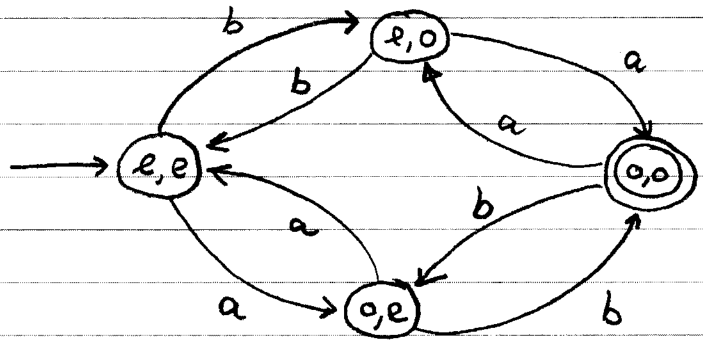
15. $L(M) = \{ w \mid w \in \{a,b\}^*, |w|_a$ and $|w|_b$ are odd $\}$

find automaton M

We need 4 states

	$\lvert w \rvert_a$	$\lvert w \rvert_b$
$S = (e,e)$	even	even
(e,o)	even	odd
(o,e)	odd	even
accepting state : (o,o)	odd	odd

zero a's and zero b's is state (e,e)



16. $L = \{ a^n b^n \mid n \geq 0 \}$

G : $S \to A$ not regular, but context free

$A \to \lambda$

$A \to aAb$

There is no regular expression that can generate L because of the pumping lemma, thus there is no regular grammar